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Asymptotics and corrections to asymptotics of non-singlet structure function at low x

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Abstract : We comment on the uniqueness of t-evolution ($t = \log(Q^2/\Lambda^2)$) of non-singlet structure functions at low x obtained from DGLAP equations Dependence of asymptotics on boundary conditions are also discussed.

Keywords : structure functions, low x, asymptotics

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In recent years, an approximate method of solving DGLAP equations [1-4] at low x has been pursued [5, 6]. In that approach, we expressed the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations as partial differential equations in x (the Bjorken variable $x = Q^2/2p.q$) and $t(t = \ln Q^2/\Lambda^2)$ using the Taylor series expansion and assuming its validity at low x. One of the limitations of the approach is that the solutions reported are not unique [5, 6]. They are selected as the simplest ones with a single boundary condition – the non-perturbative x distribution at some initial point $t = t_0$. However, complete solution of DGLAP equations with two differential variables in general, need two boundary conditions [7, 8].

The aim of the present note is to explore the uniqueness of the solution when it satisfies some given physically appropriate boundary conditions. The DGLAP equation for non-singlet structure function which evolve independent of singlet and gluon distributions [1-4] is

$$\frac{\partial F^{NS}}{\partial t} = \frac{A_f}{t}\left[(3+4\log(1-x))F^{NS}(x,t)\right.$$

$$\left.+2\int_x^1 \frac{dz}{1-z}\left[\left(1+z^2\right)F^{NS}\left(\frac{x}{z},t\right) - 2F^{NS}(x,t)\right]\right], \quad (1)$$

where $t = \log\left(\frac{Q^2}{\Lambda^2}\right)$ and $A_f = \frac{4}{33-2N_f}$; N_f being the number of quark flavours.

Let us introduce the variable $u = 1 - z$ and note that

$$\frac{x}{1-u} = x\sum_{k=0}^{\infty} u^k. \quad (2)$$

This series (2) is convergent for $u < 1$. Since $x < z < 1$, so $0 < u < 1 - x$ and hence the convergence condition is satisfied. Using (2), we wite in (1)

$$F^{NS}\left(\frac{x}{z},t\right) = F^{NS}(x,t) + \sum_{l=1}^{\infty}\frac{x^l}{l!}\left(\sum_{k=1}^{\infty}u^k\right)^l \frac{\partial^l F^{NS}(x,t)}{\partial x^l} \quad (3)$$

which covers the whole range of u, $0 < u < 1 - x$.

Non-singlet structure functions are expected to be well-behaved in the entire x range, unlike the gluon or singlet structure functions which might diverge for $x \to 0$ as in Balitsky-Fadin-Kuraev-Lipatov (BFKL) inspired models [9, 10]. It is therefore justified if the higher order derivatives *i.e.* $\frac{\partial^l F^{NS}}{\partial x^l}$ for $l > 1$ are neglected in (3). This is more justifiable for small $x(x << 1)$, yielding

$$F^{NS}\left(\frac{x}{z},t\right) = F^{NS}(x,t) + x\sum_{k=l}^{\infty}u^k \frac{\partial F^{NS}(x,t)}{\partial x}. \quad (4)$$

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asymptotic scaling [18] holds. Besides at very high x ($x \to 1$), it should also conform to the boundary conditions as given by (25) instead of asymptotic limit (26).

This then yields

$$t_0 X^{NS}\big|_{x\to 1} + \alpha F^{NS}\big|_{x\to 1} Y^{NS}(x)\big|_{x\to 1} = \beta \tag{43}$$

and

$$tX^{NS}(x)\Big|_{x\le x_0} + \alpha F^{NS}_{DLA} Y^{NS}(x)\Big|_{x\le x_0} = \beta \tag{44}$$

instead of (27) and (28).

Using (43) and (44) one finally obtains

$$F^{NS}(x,t) = \frac{\left(1 - \dfrac{t}{t_0}\dfrac{X(x)}{to\ X(x)\big|_{x\to 1}}\right)}{\left(1 - \dfrac{t}{t_0}\dfrac{X(x)\big|_{x\le x_0}}{X(x)\big|_{x\to 1}}\right)} \left[F^{NS}_{DLA}(x,t)\frac{Y(x)\big|_{x\le x_0}}{Y(x)} \right.$$

$$\left. - F(x,t_0)\big|_{x\to 1}\frac{Y(x)\big|_{x\to 1}}{Y(x)} \right] + F(x,t_0)\big|_{x\to 1}\frac{Y(x)\big|_{x\to 1}}{Y(x)} \tag{45}$$

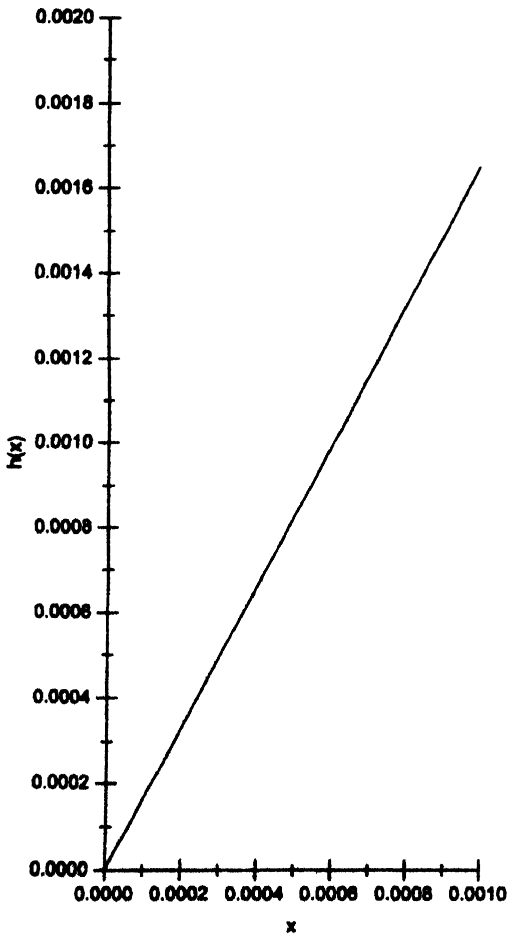

Figure 1. $h(x)$ *vs* x as defined in (33) of the text.

which reduces to $F^{NS}_{DLA}(x,t)$ for $x \le x_0$. Eq. (45) thus contains finite $-x$ corrections to the DLA result (34) in a closed form.

Let us now make a few comments. As has been mentioned earlier, the DGLAP equations have been solved earlier analytically and that too exactly in the low-x regime. Extensive numerical analysis of the exact solutions of those equations in low-x region has also been done in the last few years with Next to leading order (NLO) accuracy [19-21]. Questions about the scheme dependence of such analysises has just began [22]

In the light of such a progress, it is meaningful to conclude with the physics issue clearly brought out in the present work reported at LO level : we have shown that the asymptotics of DGLAP equation depend crucially on the boundary conditions so much that alternative asymptotics other than the DLA ones can also be obtained as have been pursued earlier [5, 6] However, if the DLA asymptotics are imposed as boundary conditions, one can have a closed expression to take into account the finite-x corrections to such asymptotics. This feature is found to be true at NLO level too [23].

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